DUTT

Theory of the Ignition Coil

Electrical Engineering

M. S.

1912

THEORY OF THE IGNITION COIL

BY

MATI LAL DUTT

B. S., University of Illinois, 1911

THESIS

Submitted in Partial Fulfillment of the Requirements for the

Degree of

MASTER OF SCIENCE

IN ELECTRICAL ENGINEERING

IN

THE GRADUATE SCHOOL

OF THE

UNIVERSITY OF ILLINOIS

1912

UNIVERSITY OF ILLINOIS

THE GRADUATE SCHOOL

May 31, 1912

I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY

MATI LAL DUTT

ENTITLED THEORY OF THE IGNITION COIL

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE

DEGREE OF MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

Ernst Berg
In Charge of Major Work

Ernst Berg
Head of Department

Recommendation concurred in:

Morgan Brooks
Ellery B. Paine
J M Bryant

Committee on Final Examination

--- CONTENTS ---

THEORY OF THE MODERN IGNITION COIL

I Introduction

The modern ignition coil, otherwise known as Induction coil, Inductorium, or Rhumkorff coil was regarded a few years ago principally as a useful laboratory apparatus and was chiefly used to illustrate the principles of electro-magnetic induction. To-day it has become an article of necessity to thousands of people, and the industry of manufacturing induction coils has become a very important one. Among the uses to which induction coils are put, may be mentioned the wireless telegraph, the excitation of X-ray tubes, igniting the explosive mixture of gas and gasoline in the internal combustion engines of various types, electric gas lighting, etc.

In spite of the extensive use of the induction coil and the trouble arising from ignorance in the handling of such coils, very little has been written regarding its principles and operation from an engineering and practical point of view. Of course, all text books on physics give an elementary explanation of its action, and a few highly mathematical articles, written more from the stand point of the mathematician than from that of the engineer, have been published. The practical engineer having ordinary mathematical knowledge, finds very little information on the subject and his knowledge is limited to the usual text books. Even

this meager information is so scattered that it is quite beyond the reach of practical engineers. The attempt of this thesis will be to present the whole subject in a logical manner and develop some equations from fundamental principles to be of use to the every day engineer.

II Physical Theory and Principles of Operation

When we examine the development of the ignition coil, we find that the evolution followed the general law "that improvements in experimental instruments advance along definite lines by a process of evolution in which rudimentary forms are successively replaced by more and more completely developed machines." Most people who are interested in the induction coil think that Faraday made the discovery of electro-magnetic induction with his rude ring transformer, but the fact is that Joseph Henry, a young teacher in the Albany Academy, was an independent discoverer of electro-magnetic induction. This is proven by an account of the manner in which he had independently performed a similar experiment in the previous autumn before receiving an account of Faraday's work.[1] But William Sturgeon, an eminent English physicist, was the first person to construct a coil which was practically the same in the general form as the induction coil of to-day. The next great improvement was that of inserting a condenser across the "break" of the primary circuit of the coil. This was rendered by Fizeau, a French physicist.[2] This application of the condenser in the primary circuit made possible the universal use of the induction coil. The modern ignition coil is essentially the same as the Rhumkorff coil or induction coil. It consists of two coils, primary and secondary, and a central iron core. The

latter is usually composed of a bundle of soft iron wires (insulated from each other by shellac). The primary coil which is wound around this core is made of a few turns of thick insulated copper wire; the secondary coil which surrounds the primary coil is made of very fine, well insulated wire of considerable length having a large number of turns.

Fig. I represents the picture of such a coil showing the details. The current flowing from the battery B through the binding screw A, the contact screw b and the hammer h, enters the primary coil, where it acts inductively on the secondary coil. Having traversed the primary coil it comes out at f, and returns to the battery through the binding screw C. When the current flows through the primary coil it magnetises the iron core which attracts the iron hammer or vibrator h, thus breaking the primary circuit at e. This interruption of the primary circuit causes the magnetic flux to die away. The hammer is then released thus completing the electric circuit, and the current flows again. This is repeated at a great frequency which depends on the inertia of the hammer and the constants of the coil. The repeated appearance and disappearance of the current in the primary coil induces an alternating electro-motive force of high voltage

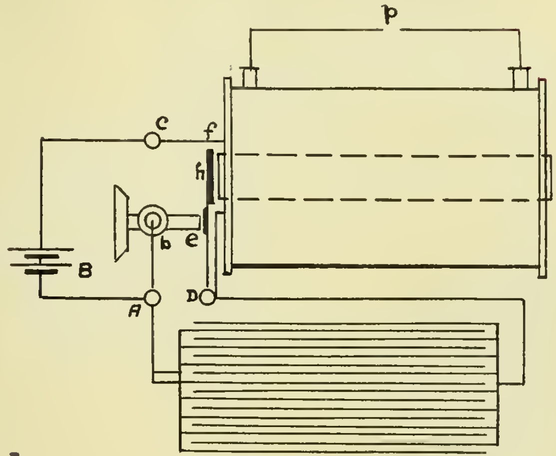

Fig.1

in the secondary coil. The strength of this electro-motive force depends on the ratio of the number of turns in the secondary coil to that of the primary coil. When the electro-motive force is large enough to overcome the resistance between the needle points at p, a spark passes.

To make the break more sudden and to set up an electrical oscillation in the circuit it is usual to put a condenser across the "break points" of the primary circuit. This condenser is thus so connected that it does not affect the current rush in making but only in breaking. Owing to the magnetic induction in the coil, the current in the primary coil cannot rise suddenly to its maximum strength. It follows the law,

$$i = \frac{E}{r}\left(1 - \epsilon^{-\frac{r}{L}t}\right)$$

where i = primary current in amperes

E = impressed electro-motive force in volts

r = resistance of primary coil in ohms

L = self induction in henries

$\epsilon = 2.718$

t = time in seconds

The current therefore requires an appreciable fraction of a second to flow through the primary coil containing inductance and resistance only. Consequently at the make of the primary circuit the rate of change of flux is relatively slow. In the case of the break, however, the primary circuit contains inductance, resistance and capacity, as a consequence the rate of change of flux in the case of break is greater than in the

case of make. Hence the maximum voltage across the terminals of the secondary coil occurs at the time of primary break.

The object of the condenser is primarily two-fold. It enables the primary circuit to be broken more or less sparklessly and it serves as a spring which absorbs the electro-magnetic energy, which is equal to $\frac{L\,i^2}{2}$, at the time of the break. This absorbed energy is sent back from the condenser as a reversed current into the primary coil causing magnetic lines to appear in the opposite direction. Thus the electro-motive force in the secondary coil is increased and an intensified spark occurs at the spark gap.

More clearly, if a condenser is connected across the "break points" of the primary circuit, the energy in the magnetic field dies down and charges the condenser for a short time; the condenser then discharges sending a reversed current through the coil, thus setting up an electrical oscillation. The electro-motive force set up in the secondary coil is then the result of the stoppage of the primary current and its immediate reversal in direction, and this is equivalent to the removal of a certain number of lines of induction from the secondary circuit, and their immediate insertion into it in the opposite direction. Hence when a condenser is used the secondary electro-motive force would be just double the electro-motive force in the case when no condenser is used (neglecting losses). "The condenser acts by setting up electrical oscillations, and it does away with the spark or largely diminishes it, in virtue of the fact that the condenser acts at

at the moment of "break" as if it were a shunt circuit of negative self-induction, only with this difference--that instead of dissipating energy like a circuit of resistance and induction it returns to the primary circuit in the form of a reversed current, and increases the total change of induction through the secondary circuit".[3]

FUNDAMENTAL EQUATIONS IN TRANSIENTS.

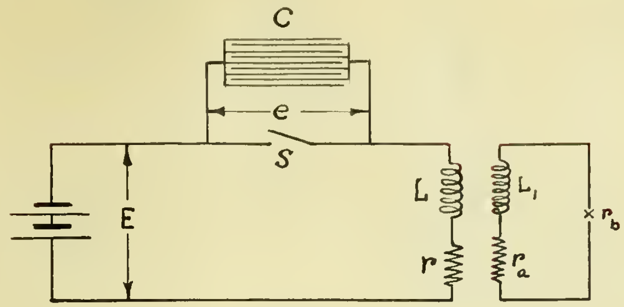

Fig. 2

The induction coil stripped of all mechanical detail may be represented diagrammatically by Fig. 2. E is the constant electromotive force impressed, L and r are the self-induction and resistance of the primary coil. C is a capacity connected across the switch or interrupter S. e is the electromotive force of the condenser. Also, L_1 is the self-induction of the secondary coil, and r_1 is the sum of the resistances, r_a of the coil, and r_b of the spark gap, and M is equal to the mutual induction of the two coils, primary and secondary.

From the fundamental law that in a closed circuit the algebraic sum of the electromotive forces is equal to zero, we find at the instant after opening the switch S, that the following equations are true.

$$ir + L\frac{di}{dt} + M\frac{di_1}{dt} + e - E = 0 \qquad - - - - - - - 1$$

$$i_1r_1 + L_1\frac{di_1}{dt} + M\frac{di}{dt} = 0 \qquad - - - - - - - 2$$

Multiplying equation 1 by L_1 and equation 2 by $-M$, we get,

$$L_1ir + LL_1\frac{di}{dt} + L_1M\frac{di_1}{dt} + eL_1 - EL_1 = 0 \qquad - - - - - - - 3$$

$$-Mi_1r - ML_1\frac{di_1}{dt} - M^2\frac{di}{dt} = 0 \qquad - - - - - 4$$

Adding equations 3 and 4 and solving for i_1, we get,

$$i_1 = \left(\frac{LL_1 - M^2}{Mr_1}\right)\frac{di}{dt} + \frac{L_1 r}{Mr_1} i + \frac{L_1}{Mr_1 c}\int i\,dt - \frac{EL_1}{Mr_1} \quad - - - - - - \quad 5$$

Differentiating,

$$\frac{di_1}{dt} = \left(\frac{LL_1 - M^2}{Mr_1}\right)\frac{d^2 i}{dt^2} + \frac{L_1 r}{Mr_1}\frac{di}{dt} + \frac{L_1}{Mr_1 c} i \quad - - - - - - - \quad 6$$

Substituting the expression for $\frac{di_1}{dt}$ in equation (1)

$$ir + L\frac{di}{dt} + M\left\{\left(\frac{LL_1 - M^2}{Mr_1}\right)\frac{d^2 i}{dt^2} + \frac{L_1 r}{Mr_1}\frac{di}{dt} + \frac{L_1}{Mr_1 c} i\right\} + e - E = 0$$

$$\text{or } \left(\frac{LL_1 - M^2}{r_1}\right)\frac{d^2 i}{dt^2} + \left(\frac{L_1 r}{r_1} + L\right)\frac{di}{dt} + \left(\frac{L_1}{r_1 c} + r\right) i + e - E = 0$$

Since $i = \frac{dq}{dt}$, it can be written as

$$\left(\frac{LL_1 - M^2}{r_1}\right)\frac{d^3 q}{dt^3} + \left(\frac{L_1 r}{r_1} + L\right)\frac{d^2 q}{dt^2} + \left(\frac{L_1}{r_1 c} + r\right)\frac{dq}{dt} + \frac{q}{c} - E = 0 \quad - - - - \quad 7$$

$$\frac{d^3 q}{dt^3} + \left(\frac{L_1 r + L r_1}{LL_1 - M^2}\right)\frac{d^2 q}{dt^2} + \frac{r r_1 c + L_1}{c(LL_1 - M^2)}\frac{dq}{dt} + \frac{q r_1}{c(LL_1 - M^2)} - \frac{E r_1}{LL_1 - M^2} = 0 \quad - - \quad 8$$

This is a differential equation of the third order whose right-hand member is zero and is solvable by means of the auxiliary equation.

The auxiliary equation is,

$$m^3 + \left(\frac{L_1 r + L r_1}{LL_1 - M^2}\right) m^2 + \left(\frac{r r_1 c + L_1}{c(LL_1 - M^2)}\right) m + \frac{r_1}{c(LL_1 - M^2)} = 0$$

let $\frac{L_1 r + L r_1}{LL_1 - M^2} = a$

$\frac{r r_1 c + L_1}{c(LL_1 - M^2)} = b$

$\frac{r_1}{c(LL_1 - M^2)} = c$

the equation then becomes:

$$m^3 + am^2 + bm + c = 0 \quad - - - - - - - \quad 9$$

Removing the second term by substituting for m an assumed unknown $y - \frac{1}{3}a$, we get the new equation:

$$y^3 + py + q = 0 \qquad 10$$

where $\quad p = -\left(\frac{a^2}{3} - b\right)$

and $\quad q = \left(\frac{2a^3}{27} - \frac{ba}{3} + c\right)$

The three roots of this cubic equation by Cardian's rule are:

$$y_1 = \sqrt[3]{-\tfrac{1}{2}q + \sqrt{\left(\tfrac{q}{2}\right)^2 + \left(\tfrac{p}{3}\right)^3}} + \sqrt[3]{-\tfrac{1}{2}q - \sqrt{\left(\tfrac{q}{2}\right)^2 + \left(\tfrac{p}{3}\right)^3}}$$

$$y_2 = \omega_1\sqrt[3]{-\tfrac{1}{2}q + \sqrt{\left(\tfrac{q}{2}\right)^2 + \left(\tfrac{p}{3}\right)^3}} + \omega_2\sqrt[3]{-\tfrac{1}{2}q - \sqrt{\left(\tfrac{q}{2}\right)^2 + \left(\tfrac{p}{3}\right)^3}}$$

$$y_3 = \omega_2\sqrt{-\tfrac{1}{2}q + \sqrt{\left(\tfrac{q}{2}\right)^2 + \left(\tfrac{p}{3}\right)^3}} + \omega_1\sqrt[3]{-\tfrac{1}{2}q - \sqrt{\left(\tfrac{q}{2}\right)^2 + \left(\tfrac{p}{3}\right)^3}}$$

wherein $\quad \omega_1 = \frac{-1 + j\sqrt{3}}{2}$

$$\omega_2 = \frac{-1 - j\sqrt{3}}{2}$$

Since

$$m = y - \frac{a}{3}$$

$\therefore$ the roots of the equation 9 are:

$$m_1 = y_1 - \frac{a}{3}$$

$$m_2 = y_2 - \frac{a}{3}$$

$$m_3 = y_3 - \frac{a}{3}$$

$\therefore$ the general solution of the differential equation is:

$$q = A_1 \varepsilon^{m_1 t} + A_2 \varepsilon^{m_2 t} + A_3 \varepsilon^{m_3 t} - EC \qquad 11$$

$$i = \frac{dq}{dt} = A_1 m_1 \varepsilon^{m_1 t} + A_2 m_2 \varepsilon^{m_2 t} + A_3 m_3 \varepsilon^{m_3 t} \qquad 12$$

$$\frac{di}{dt} = A_1 m_1^2 \varepsilon^{m_1 t} + A_2 m_2^2 \varepsilon^{m_2 t} + A_3 m_3^2 \varepsilon^{m_3 t}$$

Substituting the expressions for $\frac{di}{dt}$, i and q in equation 5, we get,

$$i_1 = \frac{LL_1 - M^2}{Mr_1}\left(A_1 m_1^2 \varepsilon^{m_1 t} + A_2 m_2^2 \varepsilon^{m_2 t} + A_3 m_3^2 \varepsilon^{m_3 t}\right)$$
$$+ \frac{L_1 r}{Mr_1}\left(A_1 m_1 \varepsilon^{m_1 t} + A_2 m_2 \varepsilon^{m_2 t} + A_3 m_3 \varepsilon^{m_3 t}\right)$$
$$+ \frac{L_1}{Mr_1}\left(A_1 \varepsilon^{m_1 t} + A_2 \varepsilon^{m_2 t} + A_3 \varepsilon^{m_3 t} - EC\right) - \frac{EL_1}{Mr_1} \text{ ---- } 13$$

The three integration constants A_1, A_2 and A_3 are found by the initial condition; at the time of closing the primary circuit at the switch or interrupter s, let $t = 0$; then

$$i = 0$$

$$i_1 = 0$$

$$e = E \quad \therefore \quad q = ec = EC$$

Substituting the values of A_1, A_2 and A_3 in equations 11, 12 and 13, the expressions for condenser charge, primary current and secondary current in terms of t can be calculated.

But as the equations are long and cumbersome, two special cases having certain modifications (though they do not exist practically) will be considered to understand the action clearly.

CASE. 1

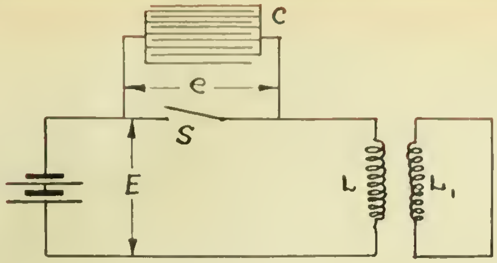

Fig. 3

Assume $r = r_1 = 0$

Fig 3 represents such a circuit having no resistance in the primary or the secondary coil. The notations are the same as in the general case. Then the equations 1 and 2 become:

$$L\frac{di}{dt} + M\frac{di_1}{dt} + e - E = 0 \quad - \quad - \quad - \quad - \quad - \qquad 14$$

$$L_1\frac{di_1}{dt} + M\frac{di}{dt} = 0 \quad - \quad - \quad - \quad - \quad - \quad - \qquad 15$$

Multiplying equation 14 by L_1 and equation 15 by $-M$ again we get,

$$LL_1\frac{di}{dt} + ML_1\frac{di_1}{dt} + eL_1 - EL_1 = 0$$

$$-ML_1\frac{di_1}{dt} - M^2\frac{di}{dt} = 0$$

Adding,

$$(LL_1 - M^2)\frac{di}{dt} + \frac{L_1}{c}q - EL_1 = 0$$

Since $i = \frac{dq}{dt}$, it can b written as before

$$\frac{d^2q}{dt^2} + \frac{L_1}{c(LL_1 - M^2)}q - \frac{EL_1}{LL_1 - M^2} = 0 \quad - \quad - \quad - \quad - \quad - \quad - \quad - \qquad 16$$

Solving this differential equation by the auxiliary equation $m^2 + \frac{L_1}{c(LL_1 - M^2)} = 0$, we find

$$q = A_1 \varepsilon^{-j\sqrt{\frac{L_1}{c(LL_1-M^2)}}\,t} + A_2 \varepsilon^{+j\sqrt{\frac{L_1}{c(LL_1-M^2)}}\,t} - Ec$$

$$= A_1 \sin\left(\sqrt{\frac{L_1}{c(LL_1-M^2)}}\,t + A_2\right) - EC \quad - \quad - \quad - \quad - \qquad 17$$

But $i = \frac{dq}{dt}$

$$\therefore \quad i = A_1\sqrt{\frac{L_1}{C(LL_1-M^2)}}\,\cos\left(\sqrt{\frac{L_1}{C(LL_1-M^2)}}\,t + A_2\right) \quad - - - - \quad 18$$

$$\frac{di}{dt} = -A_1\frac{L_1}{C(LL_1-M^2}\,\sin\left(\sqrt{\frac{L_1}{C(LL_1-M^2)}}\,t + A_2\right) \quad - - - - - \quad 19$$

Substituting equation 19 in equation 15, we get,

$$L_1\frac{di_1}{dt} + M\left\{-A_1\frac{L_1}{C(LL_1-M^2)}\,\sin\left(\sqrt{\frac{L_1}{C(LL_1-M^2)}}\,t + A_2\right)\right\} = 0$$

$$\text{or } L_1\frac{di_1}{dt} = \frac{MA_1L_1}{C(LL_1-M^2)}\,\sin\left(\sqrt{\frac{L_1}{C(LL_1-M^2)}}\,t + A_2\right)$$

$$\text{or } i_1 = -\frac{MA_1}{L_1}\sqrt{\frac{L_1}{C(LL_1-M^2)}}\,\cos\left(\sqrt{\frac{L_1}{LL_1-M^2}}\,t + A_2\right) + A_3 \quad - - \quad 20$$

The period of the primary oscillation

$$= \frac{2\pi}{\sqrt{\frac{L_1}{C(LL_1-M^2)}}}$$

$$= 2\pi\sqrt{\frac{C(LL_1-M^2)}{L_1}}$$

The same result may be obtained from the general equation 8 by putting $r = r_1 = 0$. It is:

$$\frac{d^3q}{dt^3} + \frac{L_1}{C(LL_1-M^2)}\,dq = 0$$

$\therefore$ the auxiliary equation

$m^3 + \frac{L_1}{C(LL_1-M^2)}\,m = 0$ has the following roots

$$m_1 = +j\sqrt{\frac{L_1}{C(LL_1-M^2)}}$$

$$m_2 = -j\sqrt{\frac{L_1}{C(LL_1-M^2)}}$$

$$m_3 = 0$$

$$\therefore q = A_1\,\varepsilon^{+j\sqrt{\frac{L_1}{C(LL_1-M^2)}}\,t} + A_2\,\varepsilon^{-j\sqrt{\frac{L_1}{C(LL_1-M^2)}}\,t} - EC$$

$$= A_1\,\sin\left(\sqrt{\frac{L_1}{C(LL_1-M^2)}}\,t + A_2\right) - EC$$

Integrate equation 15,

$$L_1 i_1 + M i = K \text{ (constant)} \quad - \quad - \quad - \quad - \qquad 21$$

This shows that the primary and the secondary current oscillate periodically when one is maximum the other is minimum. If the primary current had been closed for an appreciable time so that the secondary current had died down to zero, then at the time of the primary break $i_1 = 0$

$$\therefore L_1 i_1 = 0$$

hence $\quad Mi = K$

and i is the primary current which is eventually reached,

thus $\quad K = MI$

$$\therefore i_1 = \frac{K - Mi}{L_1} = \frac{MI - Mi}{L_1} = \frac{M}{L_1}(I - i) \quad - \quad - \qquad 22$$

but $i = \frac{dq}{dt} = A_1 \sqrt{\frac{L_1}{C(LL_1 - M^2)}} \, \text{Cos} \left(\sqrt{\frac{L_1}{C(LL_1 - M^2)}} \, t \right)$

Thus the primary current is also an alternating current and because we have neglected resistance it must oscillate between I and $-I$

thus for $i = -I$

$$i_1 = \frac{2M}{L_1} I$$

Comparing this value of i_1 with the secondary current as obtained in the case of the short secondary (where $i_1 = \frac{MI}{L}$), we find it is twice as large.

CASE. 2 Perfect Mutual Induction

When $M^2 = LL_1$ or $M^2 - LL_1 = 0$

To avoid need-less mathematical repetition, it would be advantageous to substitute $M^2 - LL_1 = 0$ in the general equation 7, it is:

$$\left(\frac{L_1 r}{r_1} + L\right)\frac{d^2q}{dt^2} + \left(\frac{L_1}{r_1 c} + r\right)\frac{dq}{dt} + \frac{q}{c} - E = 0$$

$$\text{or} \quad \frac{d^2q}{dt^2} + \frac{L_1 + r r_1 c}{c(L_1 r + L r_1)}\frac{dq}{dt} + \frac{r_1}{c(L_1 r + L r_1)}q - \frac{E r_1}{L_1 r + L r_1} = 0 \quad - \ 23$$

The auxiliary equation of this differential equation is,

$$m^2 + \frac{L_1 + r r_1 c}{c(L_1 r + L r_1)}m + \frac{r_1}{c(L_1 r + L r_1)} = 0 \quad - \quad - \quad - \quad 24$$

The roots are:

$$m_1 = -\frac{r r_1 c + L_1}{(L r_1 + L_1 r)c} + \sqrt{\left(\frac{r r_1 c + L_1}{2(L r_1 + L_1 r)c}\right)^2 - \frac{r_1}{(L r_1 - L_1 r)c}}$$

$$m_2 = -\frac{r r_1 c + L_1}{(L r_1 + L_1 r)c} - \sqrt{\left(\frac{r r_1 c + L_1}{2(L r_1 + L_1 r)c}\right)^2 - \frac{r_1}{(L r_1 - L_1 r)c}}$$

By inspection, we see the expression under the radical sign is positive, therefore the roots m_1 and m_2 are real.

$$\therefore q = -Ec + A_1\varepsilon^{+m_1 t} + A_2\varepsilon^{m_2 t} \quad - \quad - \quad - \quad 25$$

This shows that with perfect mutual induction the condenser charge follows an exponential curve and is not an oscillating phenomenon. Consequently i (which is equal to $\frac{dq}{dt}$) is not an oscillating curve.

NUMERICAL EXAMPLES

To illustrate the use of the equations previously deduced, the constants of an induction coil were determined for numerical application. The resistances of both the primary and the secondary coils were determined by means of a resistance bridge which is familiar to every electrical engineer. The method used in measuring the inductance was the well known impedance method. Each of the primary and the secondary coils was connected as shown in Fig. 4. An alternating electro-motive force was impressed on the circuit. The total current in the coil and the voltmeter was measured in each case, and this current was later corrected by deducting the current taken by the voltmeter. The current flowing through the voltmeter was determined by the equation $I_v = \frac{E}{r_v}$ where r_v is the voltmeter resistance, and E is the impressed electro-motive force. The frequency of the circuit was also determined by a frequency meter. The impedance of the coil $Z = \frac{E}{I}$ therefore the reactance, $x = \sqrt{Z^2 - R^2}$ where R = the resistance of the coil, the self-induction $L = \frac{x}{2\pi f}$ where f = the frequency of the electro-motive force.

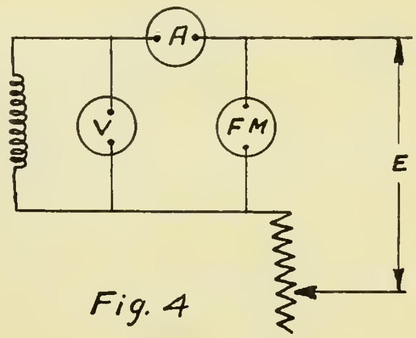

Fig. 4

The unknown capacity was determined by comparing the deflection caused by its full charge in a galvanometer with the deflection caused by a known capacity charged with same voltage in the same galvanometer. The connections are shown in Fig. 5. G is a galvanometer. The unknown capacity was charged by the battery and then discharged into the galvanometer circuit and the

deflection noted. Then the standard capacity of a known value was connected in place of C_x and the deflection was noted again. Then

$$\frac{C_x}{C_s} = \frac{d_x}{d_s}$$

where d is the deflection of the galvanometer and C the capacity.

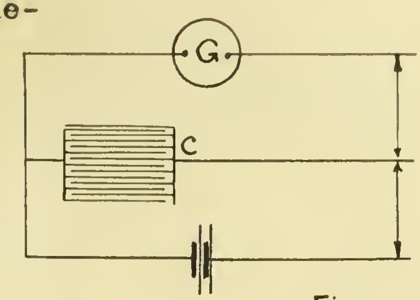

Fig. 5

The following constants were determined:

the primary resistance r = .1696 ohms

the primary reactance x = .612 ohms

the secondary resistance r_1 = 3060 ohms

the secondary reactance x_1 = 6580 ohms

the capacity C = .122 microfarads

the primary inductance L = .0017 henry

the secondary inductance L_1 = 17.5 henries

and the mutual inductance $M = \sqrt{.75\, L\, L_1}$

$= .1495$ (assumed)

General Case.

Substituting these constants of the coil under invesitgation, previously determined, in the solution of the general differential equation 8 and the auxiliary equation 9.

$$a = \frac{L_1 r + L r_1}{L L_1 - M^2} = \frac{17.5 \times .1695 + .0017 \times 3060}{.00745}$$

$$= 1096$$

$$b = \frac{r r_1 C + L_1}{C(L L_1 - M^2)} = \frac{.1696 \times 3060 \times .122 \times 10^{-6} + 17.5}{.122 \times 10^{-6} \times .00745}$$

$$= 19.22 \times 10^{9}$$

$$C = \frac{r_1}{c(LL_1 - M^2)} = \frac{3060}{.122 \times 10^{-6} \times .00745} = 33.6 \times 10^{11}$$

Substituting these values of a, b, and c in the expressions for p and q, we get,

$$p = -\left(\frac{a^2}{3} - b\right) = 19.22 \times 10^9$$

$$q = \left(\frac{2a^3}{27} - \frac{ba}{3} + c\right) = -3.67 \times 10^{12}$$

Substituing again these values of p and q in y_1, y_2, and y_3, we get,

$$y_1 = 300$$

$$y_2 = -150 + j\,138000$$

$$y_3 = -150 - j\,138000$$

but $m = y - \frac{a}{3}$ and $\frac{a}{3} = \frac{1096}{3} = 365$

$$\therefore\ m_1 = y_1 - 365 = -65$$

$$m_2 = y_2 - 365 = -515 + j\,138000$$

$$m_3 = y_3 - 365 = -515 - j\,138000$$

Therefore substituting these values of m_1, m_2, and m_3 in equation 11, we have,

$$q = -.61 \times 10^{-6} + A_1 \varepsilon^{-65t} + A_2 \varepsilon^{(-515 + j\,138000)t} + A_3 \varepsilon^{(-515 - j\,138000)t}$$

$$= -.61 \times 10^{-6} + A_1 \varepsilon^{-65t} + A_2 \varepsilon^{-515t} \left\{ \operatorname{Sin}(138000 + A_3) \right\} \quad ---\ 26$$

But $i = \frac{dq}{dt}$

$$\therefore i = -65 A_1 \varepsilon^{-65t} + A_2 \varepsilon^{-515t} 138000 \left[\mathrm{Cos}\,(138000t + A_3) \right]$$
$$- A_2 \varepsilon^{-515t} 515 \left[\mathrm{Sin}\,(138000t + A_3) \right] \quad -- 27$$

From equation 5

$$i_1 = \frac{di}{dt}\left(\frac{LL_1 - M^2}{Mr_1}\right) + \frac{L_1 r}{Mr_1} i + \frac{L_1}{Mr_1 C} q - \frac{EL_1}{Mr_1}$$

But $\frac{LL_1 - M^2}{Mr_1} = 16 \cdot 3 \times 10^{-6}$

$\frac{L_1 r}{Mr_1} = .00649$

$\frac{L_1}{Mr_1} = .03825$

$\frac{EL_1}{Mr_1} = .03825 \times 5 = .19125$

and substituting the expressions for $\frac{di}{dt}$, i (equation 27), and q (equation 26) in the above equation, it is found,

$$i_1 = 16.3 \times 10^{-6} \left[\overline{65}^2 A_1 \varepsilon^{-65t} + A_2 \varepsilon^{-515t} \left\{ 19050 \times 10^6 \,\mathrm{Sin}\,(138000t + A_3) \right.\right.$$
$$\left.\left. - 142.3 \times 10^6 \,\mathrm{Cos}\,(138000t + A_3) \right\} \right] + .00649 \left[-65 A \varepsilon^{-65t} \right.$$
$$\left. + A_2 \varepsilon^{-515} \left\{ 138000 \,\mathrm{Cos}\,(138000t + A_3) - 515 \,\mathrm{Sin}\,(138000t + A_3) \right\} \right]$$
$$+ \frac{.03825}{.122 \times 10^{-6}} \left[-.61 \times 10^{-6} + A_1 \varepsilon^{-65t} + A_2 \varepsilon^{-515t} \left\{ \mathrm{Sin}\,(138000 + A_3) \right\} \right]$$
$$- .19125$$

Simplifying,

$$i_1 = .313 \times 10^6 A_1 \varepsilon^{-65t} + 623000 A_2 \varepsilon^{-515t} \,\mathrm{Sin}\,(138000t + A_3)$$
$$- 1430 A_2 \varepsilon^{-515} \,\mathrm{Cos}\,(138000 + A_3) - .3825 \quad - - \quad 28$$

The three equations 26, 27, and 28 are the fundamental equations of the coil under investigation.

At the time of closing the switch S', Fig. 2 and letting $t=0$, we know $i=0$; $i_1=0$; and $q=EC$. Therefore the equations 26, 27, and 28 become:

$$-.61\times10^{-6} + A_1 + A_2 \sin A_3 = .61\times10^{-6}$$

$$-65A_1 + A_2\,138000 \cos A_3 - A_2\,515 \sin A_3 = 0$$

$$.313\times10^{6}A_1 + 623000\,A_2 \sin A_3 - 1430\,A_2 \cos A_3 - .3825 = 0$$

Solving these equations for A_1, A_2, and A_3 and substituting their values in the equations 26, 27, and 28, the condenser charge, the primary current, and the secondary current could be plotted with respect to time.

But since we are not concerned with the phase relation at the time of closing the switch, and also to avoid cumbersome mathematical detail, let the phase angle A_3 be assumed to be equal to 0 radians (for convenience)

Therefore from equations 26 and 27, we have

$$q = -.61\times10^{-6} + A_1\varepsilon^{-65t} + A_2\varepsilon^{-515t}\cos(138000t) \quad --- \quad 29$$

and $$i = -65A_1\varepsilon^{-65t} + A_2\varepsilon^{-515t}\,138000 \sin(138000t)$$

$$- A_2\varepsilon^{-515t}\,515 \cos(138000t) \quad --- \quad 30$$

In this case there are two unknown quantities to solve and we have two equations. Also the secondary voltage and current are directly proportional to the number of turns

and could be calculated from the ratio of transformation. Hence the expressions for finding the secondary current i_1 could be neglected in the following calculations for convenience.

At the time of closing the primary circuit at "break points", let time $t=0$; Then $i=0$ $q = EC = .61 \times 10^{-6}$

Then from equations 29 and 30, we have,

$$-.61 \times 10^{-6} + A_1 + A_2 = .61 \times 10^{-6}$$

$$-65 A_1 - A_2\, 515 = 0$$

Solving for A_1 and A_2,

$$A_1 = 1.398 \times 10^{-6}$$

$$A_2 = -.1762 \times 10^{-6}$$

Subtituting these values of A_1 and A_2 in equations 29 and 30, we find,

$$q = -.61 \times 10^{-6} + 1.398 \times 10^{-6} \varepsilon^{-65t} - .1762 \times 10^{-6} \varepsilon^{-515t} \cos(138000t) \quad 31$$

$$i = -91 \times 10^{-6} \varepsilon^{-65t} + 24300 \times 10^{-6} \varepsilon^{-515t} \sin(138000t)$$

$$+ 90.8 \times 10^{-6} \varepsilon^{-515} \cos(138000t) \text{ - - - } 32$$

also $e = \frac{q}{C}$

$$= -5 + 11.44\, \varepsilon^{-65t} - 1.445\, \varepsilon^{-515t} \cos(138000t) \text{ - - - } 33$$

In Fig. 6 are given two curves, q is the condenser charge and i is the primary current after break.

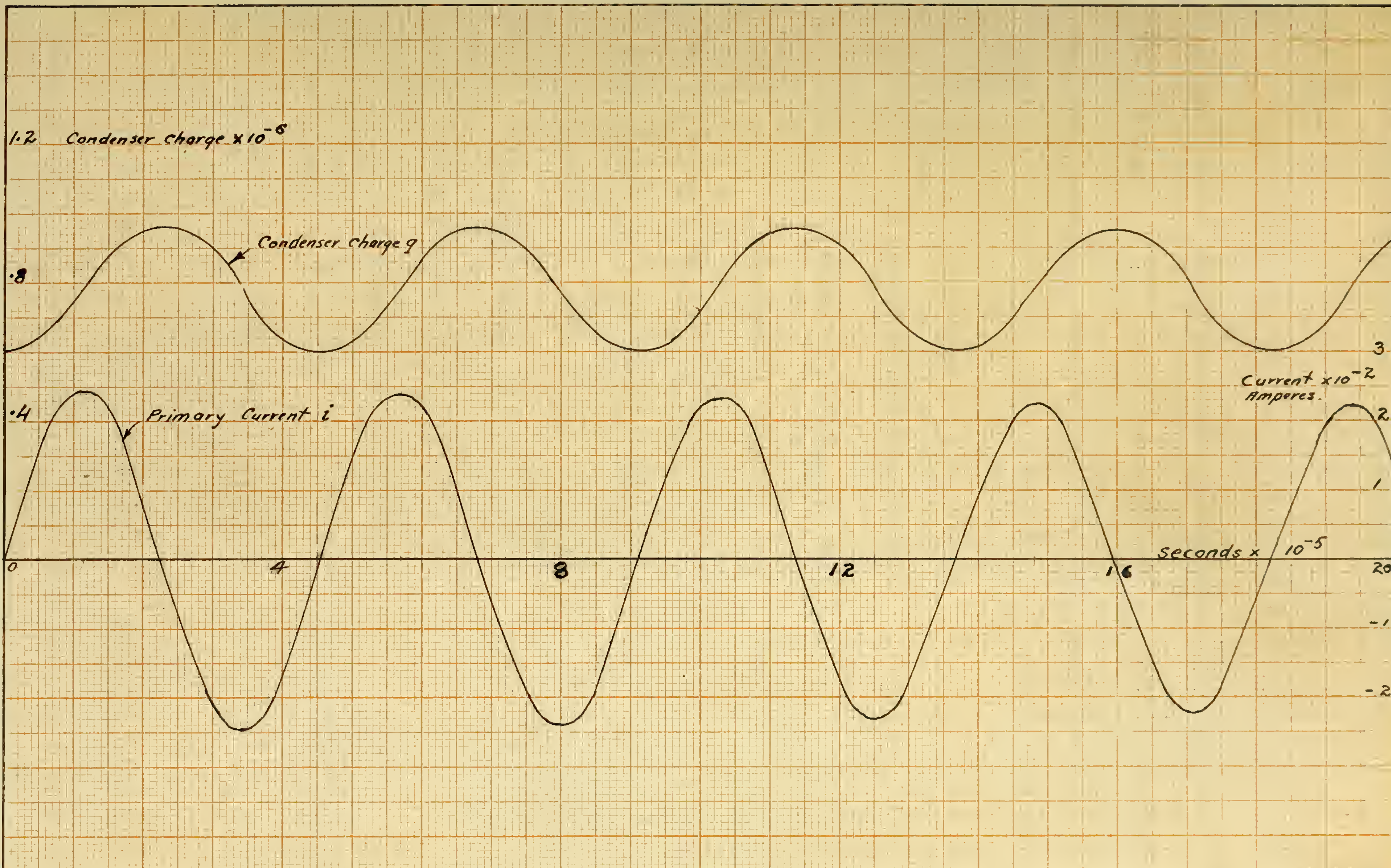
1.2
Condenser charge x10^-6
Condenser charge q
.8
.4
Primary Current i
3
Current x10^-2
Amperes.
2
1
-1
-2
Seconds x 10^-5
0
4
8
12
16
20

Fig. 6

CASE 1.

Substituting the constants of the coil, previously determined, in the first case, we find from equations 17, 18, and 20 the expressions for the Condenser charge, the primary current, and the secondary current for the coil under investigation. They are:

$$q = A_1 \operatorname{Sin}(138800t + A_2) - .61 \times 10^{-6} \quad - \quad - \quad - \quad 34$$

$$i = A_1\, 138800 \operatorname{Cos}(138800t + A_2) \quad - \quad - \quad - \quad - \quad 35$$

$$i_1 = A_1\, 2370 \operatorname{Cos}(138800t + A_2) + A_3 \quad - \quad - \quad - \quad - \quad 36$$

To find the integration constants, we know that at the time of closing the primary circuit time $t = 0$; $i = 0$, $i_1 = 0$, and $q = EC = .61 \times 10^{-6}$; hence, we have,

$$A_1 \operatorname{Sin} A_2 = 1.22 \times 10^{-6}$$

$$A_1\, 138800 \operatorname{Cos} A_2 = 0$$

$$A_1\, 2370 \operatorname{Cos} A_2 + A_3 = 0$$

Solving these equations,

$$A_1 = 1.22 \times 10^{-6}$$

$$A_2 = \frac{\pi}{2} \text{ radians}$$

$$A_3 = 0$$

Substituting these values of A_1, A_2, and A_3 in equations 34, 35, and 36, we get,

$$q = 1.22 \times 10^{-6} \operatorname{Sin}\left(138800t + \frac{\pi}{2}\right) - .61 \times 10^{-6} \quad - - - \quad 37$$

$$i = .1694 \operatorname{Cos}\left(138800t + \frac{\pi}{2}\right) \quad - - - \quad 38$$

$$i_1 = -.002995 \operatorname{Cos}\left(138800t + \frac{\pi}{2}\right. \quad - - - \quad 39$$

also $e = \frac{q}{c}$

$$= 10 \operatorname{Sin}\left(138800t + \frac{\pi}{2}\right) - 5 \quad - - - - - - \quad 40$$

In Fig. 7 the curves are shown, q reprents the condenser charge, i the primary current and i_1 the secondary current.

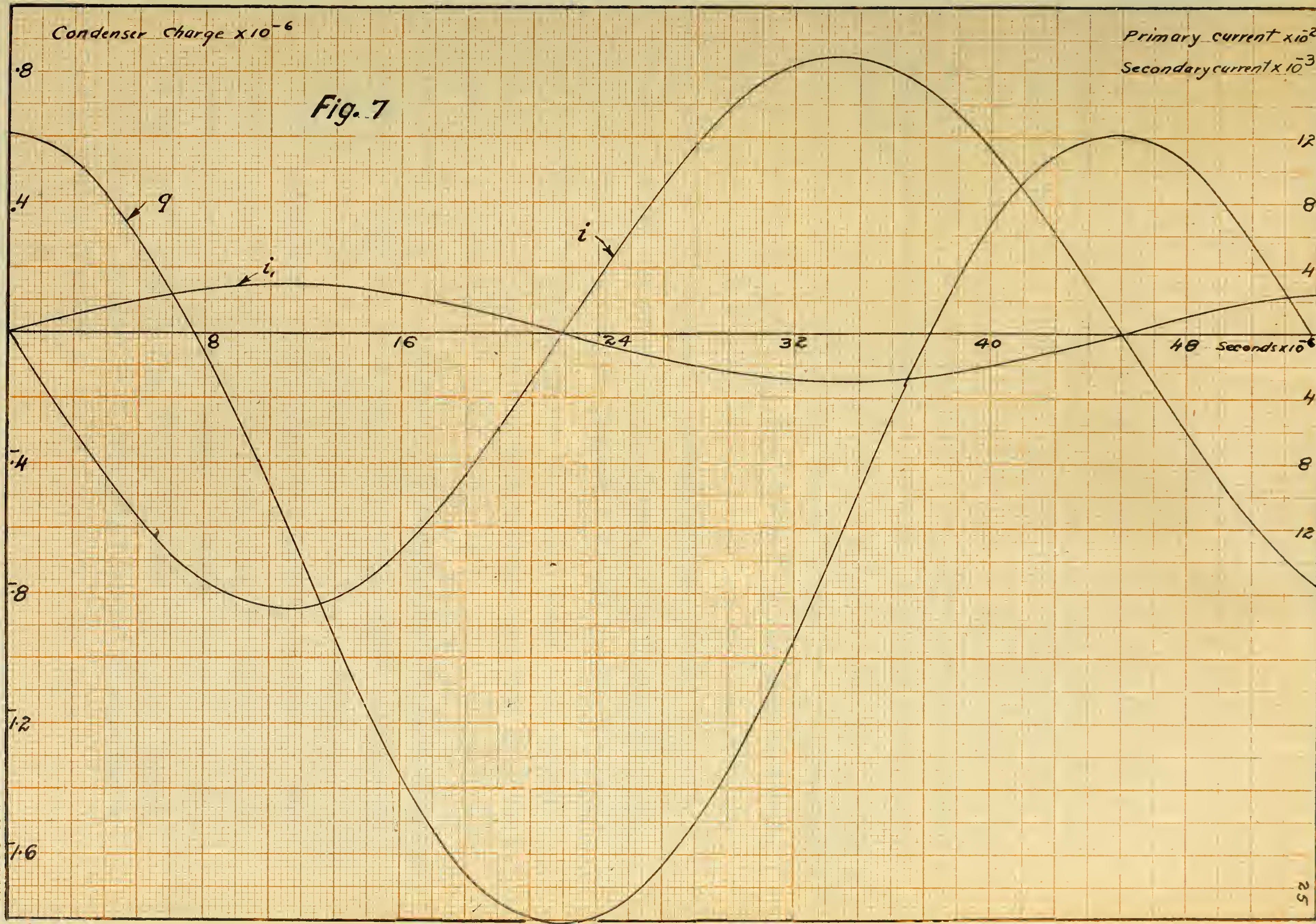
Condenser Charge x10^-6
Primary current x10^2
Secondary current x10^-3
q
i
i,
.8
.4
.4
.8
1.2
1.6
8
16
24
32
40
48
Seconds x10^-6
12
8
4
4
8
12
25

Fig. 7

VI Conclusions

The curve q in Fig. 6 shows the condenser charge after the primary circuit is broken at switch S Fig. 2. The constant term $-.61 \times 10^{-6}$ in equation 31 represents the normal condenser charge corresponding to the impressed electro-motive force. The two transient terms show how the condenser charge varies from instant to instant. The vector sum of these three terms is represented by the curve q. The electrical oscillation is started by the energy stored up in the magnetic field around the coil. At the instant of the primary break, this energy is dissipated as a contact spark if no condenser is used. But it charges a condenser when it is connected across the "break points," and consequently a current flows in the system because the time-rate of the condenser charge is the current flowing in the circuit. Hence this energy charges the condenser to a higher voltage than the impressed constant electro-motive force. This electrical instability cannot exist and the condenser discharges and sends a reversed current through the coil which sets up a magnetic field almost as strong as the original field, if we neglect the losses in the resistance and the imperfect mutual induction. The energy stored in the magnetic field thus set up charges the condenser again which discharges and sends current through the coil again and so on. Thus the electrical oscillation is set up whose period is a very small fraction of a second and generally depends on the constants of the circuit.

If we refer to the simpler case, when $r_1 = r = o$, we find from equation 17, the period of oscillation equal to

$2\pi\sqrt{\frac{(LL_1 - M^2)C}{L_1}}$. In a particular coil the terms L, L_1, and M remain constant. The deduction is that the period of oscillation for the coil is directly proportional to the square-root of the capacity. Therefore the frequency is inversely proportional to the square-root of the capacity. Hence smaller the capacity, the larger the frequency.

The period of the mechanical oscillation of the vibrator of the induction coil is much larger than the period of the electrical oscillation. As soon as the hammer is attracted, the primary circuit is broken and the electrical oscillation, previously described, starts and goes on for some time before the hammer completes the circuit again. It is a physical impossibility for a mechanical vibrator to have the same period of oscillation as that of an electrical circuit of this nature, therefore a series of reversals of magnetic lines takes place in the coil and several sparks pass between the terminals of the secondary coil during one break and make of the primary circuit.

It might also happen that when the current dies down in the act of charging the condenser, for the first time, the hammer is released. But before it goes a reasonable distance in the space between the attracted position of the hammer and the contact screw, the reversed current from the condenser magnetises the iron core in the coil. This magnetised iron

iron core, though of reversed polarity, reattracts the hammer to a certain extent and dampens or hinders its oscillation and keeps it longer in the space. The reversed current dying down charges the condenser and the hammer is released again and so on.

This process continually repeated keeps the hammer in space while sparks pass between the terminals of the secondary coil, until the strength of the current becomes so small that it cannot hold the hammer any longer in space. Then the hammer recompletes the primary circuit and some more current flows through it, at the same time short-circuiting the condenser. This phenomenon is possible because the frequency of this oscillation is so great. Thus what happens is almost equivalent to holding the hammer in space. This is not noticeable when the frequency is great. But by increasing the capacity, the frequency becomes smaller and it is noticed that the hammer becomes sluggish. The action, explained above, causes it as is evident from the discussion.

The most interesting feature of this phenomenon is that the primary coil takes a fresh charge of energy only when the oscillation cannot continue while working in unison with the other mechanical factors involved in the operation.

The term $.1762 \times 10^{-6} \ \varepsilon^{-515\,t} \cos (138000\ t)$ of equation 31 represents the amount of the condenser charge which causes the oscillation and $\frac{\pi}{138000} = .000046$ second is its period. It is also interesting to note that the period being .000046 of a second, the exponential terms ε^{-515t}, ε^{-65t}

decrease very slowly and therefore the oscillations practically retain their initial strength for a large number of reversals of magnetic lines.

All this is true when ideal conditions are obtainable, but such conditions do not exist in practice. The break of the primary current is always accompanied by a spark even when a condenser is connected across the "break points". Thus a part of the energy $\frac{L\, i^2}{2}$ is lost. The oscillation therefore starts with a smaller value of q and i and dies down faster than the theoretical curves.

Almost all ignition coils, which are seen in the market today, are different modifications of the induction coil. In the ignition of the explosive mixtures of gas engines, the battery circuit is connected to a mechanical device mounted on the engine support. Fig. 8 shows the simplest arrangement of such a device. In the commercial types a large variety

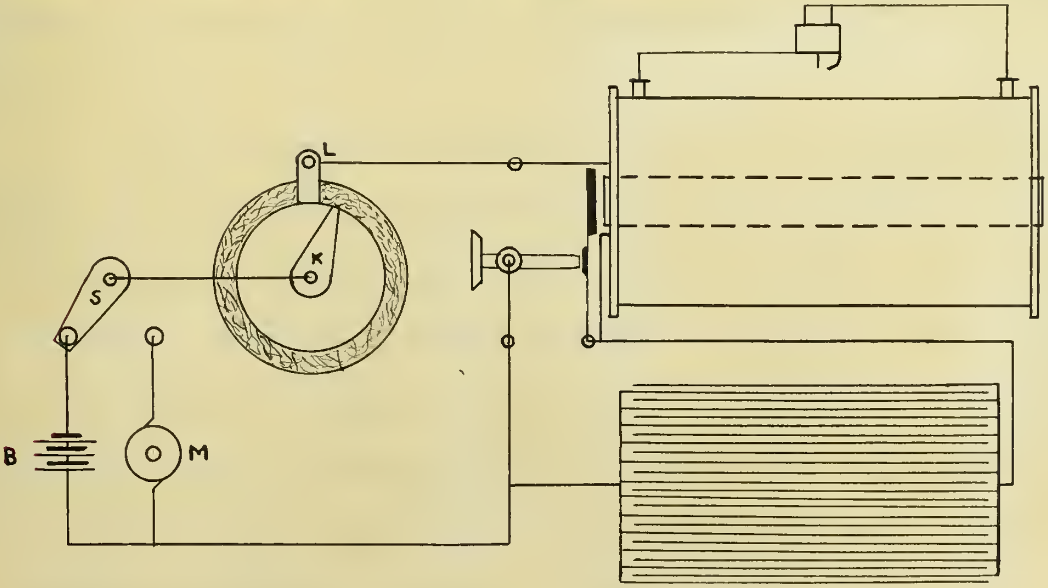

Fig. 8

of arrangements is found, and the connections are made in many ways. The switch S could be thrown either on the battery B or magneto M for the electrical current. The contact-maker K fixed on the shaft revolves and makes the electrical circuit complete once every revolution at the point L. This making of the circuit sends a current thru the primary coil, and the action previously described takes place in the coil and a series of sparks passes in the spark plug and ignites the compressed gas. Since the oscillation in an average coil is very fast, even a very small contact point at L permits a large number of electrical oscillations to take place and a large number of reversals of magnetic lines occur causing a series of sparks to pass between the points of the spark plug.

If the contact-point L is large and the contact maker continues to be in a position to send current through the primary coil after the gas is exploded, the sparks continue to pass in the spark plug and an unnecessary waste of energy occurs.

To sum up the different points regarding the condenser, we find that it minimises the contact spark at the break of the primary circuit, it sets up the electrical oscillation and causes a reversed current to flow in the primary circuit thus doubling the change in the magnetic field consequently inducing a double electro-motive force in the secondary coil. It also controlls the period of the hammer by retracting it by sending a reversed current. This controlling of the hammer allows more time for the oscillations to continue and

to permit sparks to pass between the terminals of the secondary coil.

References Quoted.

1. Sullivan's American Jornal of Science --- July 1832
2. Comptes Rendus --- Volume XXXVI --- p. 418
3. Alternating Current Transformer --- Fleming.

Bibliography

1. Internal Combustion Engines --- H. E. Wimperis
2. Electric Ignition for Gas Engine --- J. A. Newman
3. Induction Coil -- action of condenser in Rhunkorff's Coil -- Electrician, London, November 11, 1910
4. Secondary current of the induction coil,-- oscillograph by Clyde Snook -- Journal Franklin Institute, Oct. 1907
5. Induction Coils -- B. F. Bailey -- Electrical World April 14 and 21, 1910
6. The Design of Induction Coils -- W. M. O. Eddy -- Electrical World -- Dec. 26, 1906; Jan. 5, Feb. 2, 1907
7. Induction Coil in Practical Work -- Lewis Wright
8. The Design and Operation of Spark Coils -- F. W. Springer Electrical World, December 14, 1907
9. Heating Effect of the Electric Spark -- H. A. Perkins Electrical World, March 24, 1906
10. The Science of Jump Spark Coil -- J. A. Williams -- Gas Engine, July 1909
11. Electric Ignition -- Jones